Minimum Wages and the Joint Distribution Employment and Wages

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Motivation (1)

- Lots of studies on employment effects of minimum wages (Card-Krueger, Neumark-Wascher, Baker-Benjamin-Stanger, etc.)
- Standard approach in most recent studies: difference-in-differences (variation across states/provinces and time)
- Impact on (average) wages mostly viewed as a first-stage / specification check:
  - Employment effects unlikely to be credible unless there is also an effect on wages
- But looking at averages only ignores a lot of important information:
  - The effect of the minimum wage should be concentrated at the bottom end of the distribution
  - Looking at where in the distribution employment effects are concentrated provides an additional specification check
  - If high-wage workers are affected but low-wage workers are not, this likely suggests a spurious effect of the minimum wage

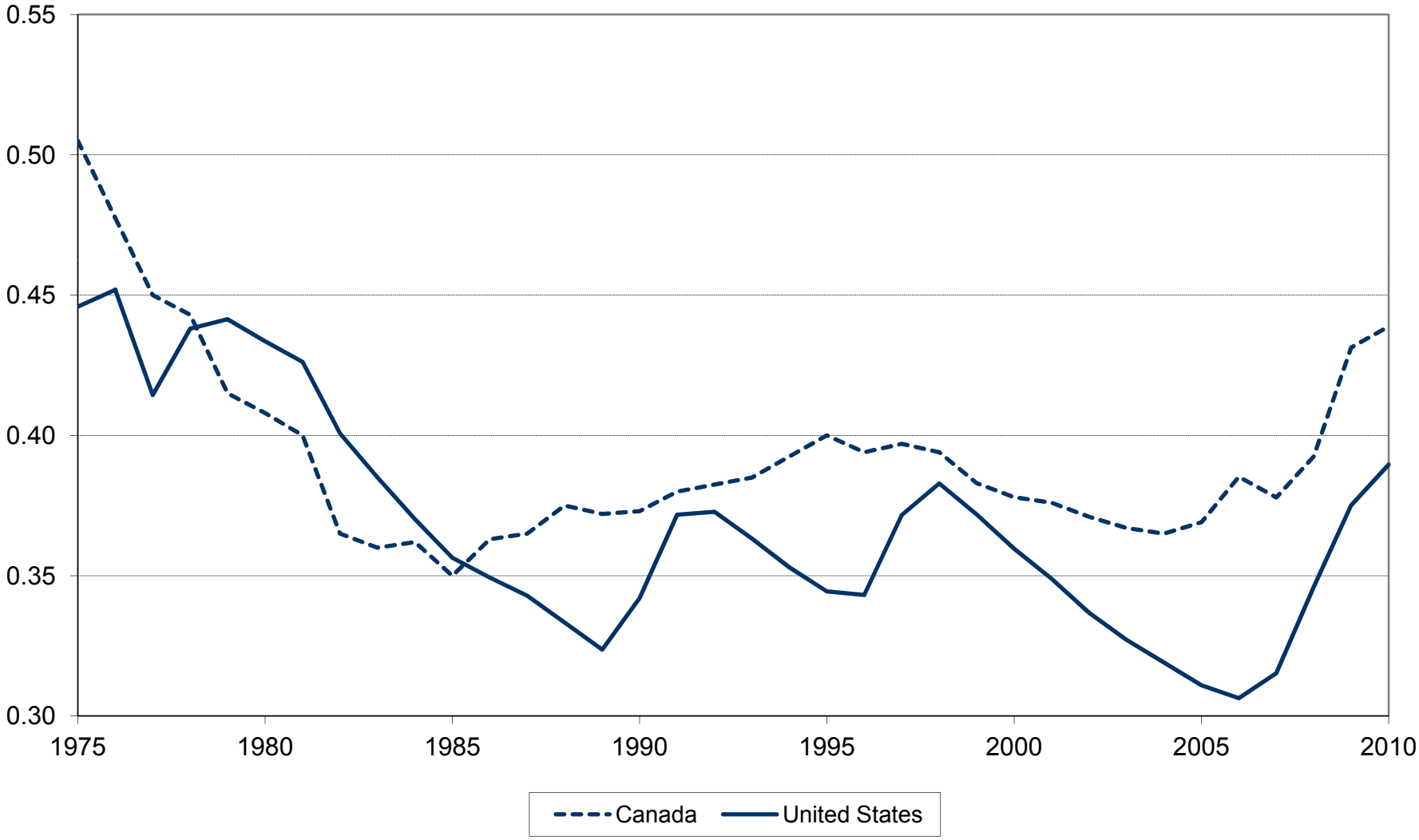
Motivation (2)

- A smaller literature has looked at the effect of the minimum wage on the wage distribution conditional on working (DiNardo-Fortin-Lemieux (1996), Lee (1999), Autor-Manning-Smith (2009))
- Suggests a large effect at the bottom end, a spike at the minimum wage, and some limited spillover effects
- But most studies look at employment and wage distribution effects in isolation
- Exceptions include Meyer and Wise (1983), Doyle (2006), but these studies are not framed in a diff-in-diffs design
- Main contribution of the paper is to propose an integrated approach for jointly modelling the employment and wage distribution in a diff-in-diffs context
- Approach is based on proportions (as in employment studies) instead of quantiles (as in wage distribution studies)

Paper in a nutshell

- Propose a “distribution regression” approach as a way of modelling the effect of the minimum wage on the scaled survivor function of wages
- Show the implication of standard models (truncation, spike, spillovers, etc.) on the scaled survivor to help interpret the empirical results
- Apply the approach the case of Canada for 1997-2010
  - Wage data available in the LFS since 1997
  - Minimum wage completely set at the provincial level
  - Ideal “diff-in-diffs” setting
  - Recent period interesting since the minimum wage hasn’t been that high since 1978
  - Find some modest disemployment and distributional effects (spike) for teenagers, but little impact for young adults

Appendix Figure A1: The Ratio of Minimum Wages to Average Wages, Canada and the United States, 1975-2010



Basic setup

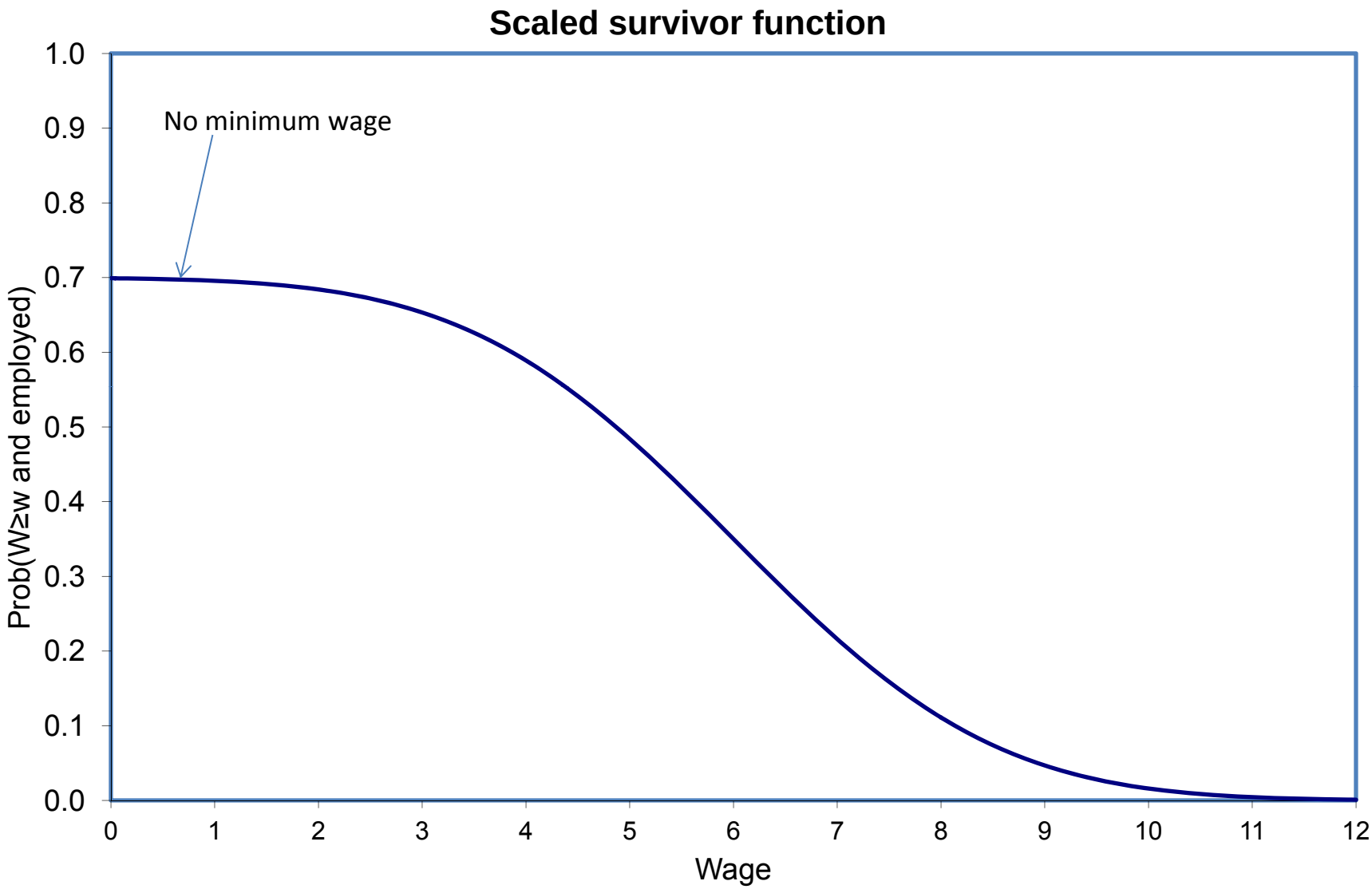
- The joint probability of being employed (E=1) and earning a wage of at least w is:

  $\Pr(E=1 \text{ and } W \geq w) = \Pr(E=1)(1-F(w|E=1))$

- The survivor function is one minus the CDF, i.e. $1-F(w|E=1)$
- It follows that $\Pr(E=1 \text{ and } W \geq w)$ is the rescaled survivor function $RS(w)$:

  $RS(w) = \Pr(E=1)(1-F(w|E=1))$

- This provides a convenient way of modelling the joint effect of the minimum wage on the distribution of wages and employment
- This is illustrated in four specific cases corresponding to common “models” for the effect of the minimum wage on employment and wages.


Scaled survivor function
No minimum wage
Prob(W≥w and employed)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
11
12
Wage

Four cases

- Case 1: “Truncation” model
  - Simple competitive model where people are paid their marginal product. They lose their job if the minimum wage goes above their productivity
- Case 2: “Spike” model
  - There are rents or labor market imperfections (e.g. search frictions) so that wages may be lower than productivity.
  - Wages may be pushed up to a higher minimum wage without resulting into employment losses
- Case 3: Spike and disemployment effects
  - Even in the presence of rents or imperfections, once the minimum wage exceeds productivity the worker will lose her job
- Case 4: Spillover effects
  - For various reasons the minimum wage may also have a positive effect on the wages of people who earn more than the new minimum.

Case 1: Minimum wage truncates the wage distribution

Let P_0 be the probability of working in absence of the minimum wage (P_0=Pr(E=1, no minimum)) .

Let F_0 (w) be the conditional distribution of wages in absence of the minimum wage [F_0(w) = F(w|E=1, no minimum))

All workers who would have earned less than the minimum wage W_m in absence of the minimum wage now lose their jobs.

It follows that the rescaled survivor function RS(w) is given by:

RS(w) = P_0(1-F_0(W_m)) if $w \leq W_m$, and
RS(w) = P_0(1-F_0(w)) if $w > W_m$.

Case 1: Effect of an increase in the minimum wage

The effect of raising the minimum wage from W_m to W_m' on the rescaled survivor function is equal to:

(1a) $P_0(1-F_0(W_m')) - P_0(1-F_0(W_m))) = P_0(F_0(W_m) - F_0(W_m')) < 0$ if $w \leq W_m$,

(1b) $P_0(1-F_0(W_m')) - P_0(1-F_0(w))) = P_0(F_0(w) - F_0(W_m')) < 0$

if $W_m < w \leq W_m'$,

(1c) 0 if $w > W_m'$.

Figure 1a: Density of wages for the truncation model

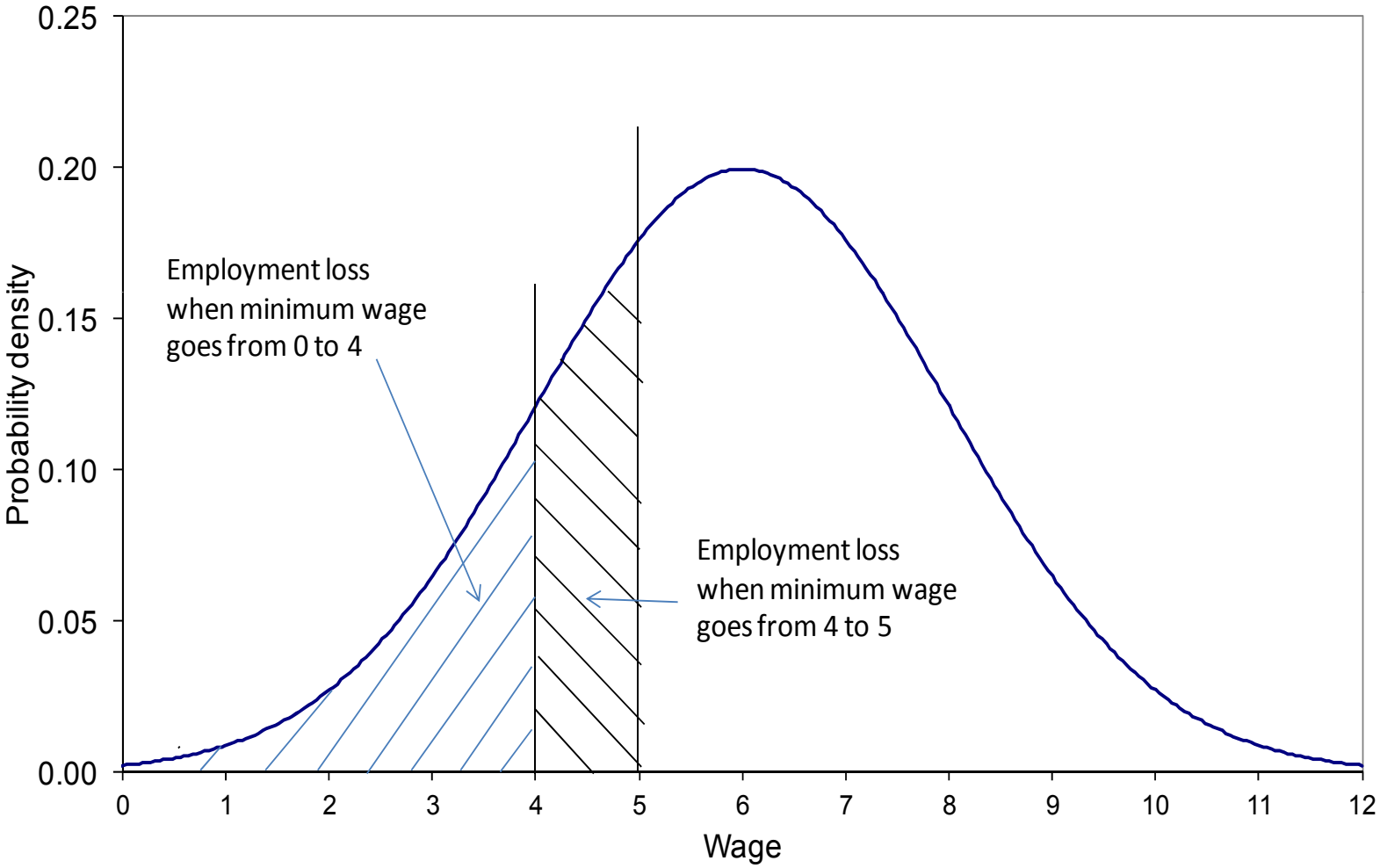
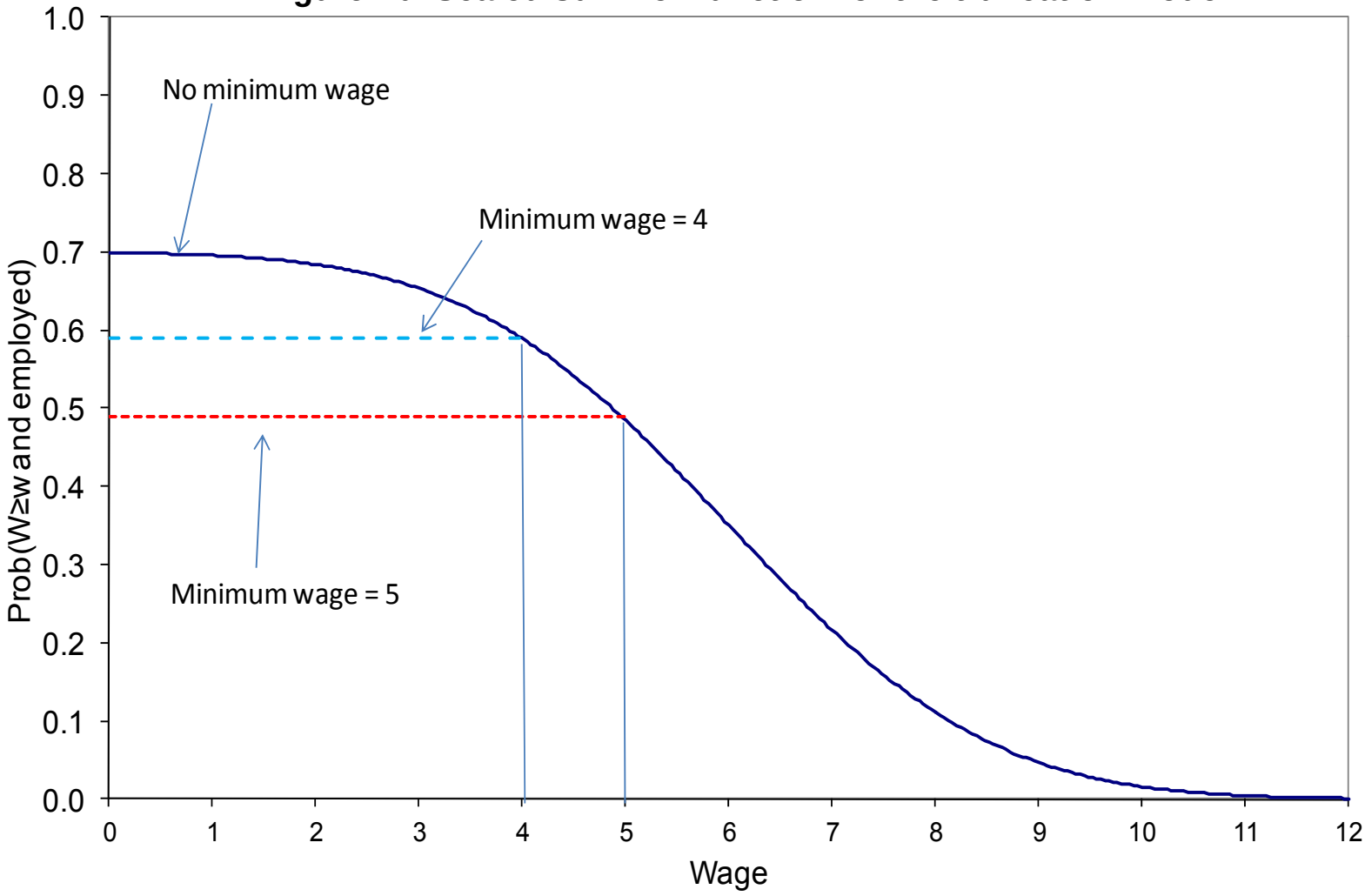


Figure 1b: Scaled survivor function for the truncation model



Case 2: Effects of the minimum wage in the pure "spike' model

Effect of raising the minimum wage from W_m to W_m' on the rescaled survivor function:

(2a) 0 if $w \leq W_m$,

(2b) $P_0(1-F_0(0)) - P_0(1-F_0(w))) = P_0F_0(w) > 0$ if $W_m < w \leq W_m'$,

(2c) 0 if $w > W_m'$.

Figure 2a: Density of wages for the pure spike model

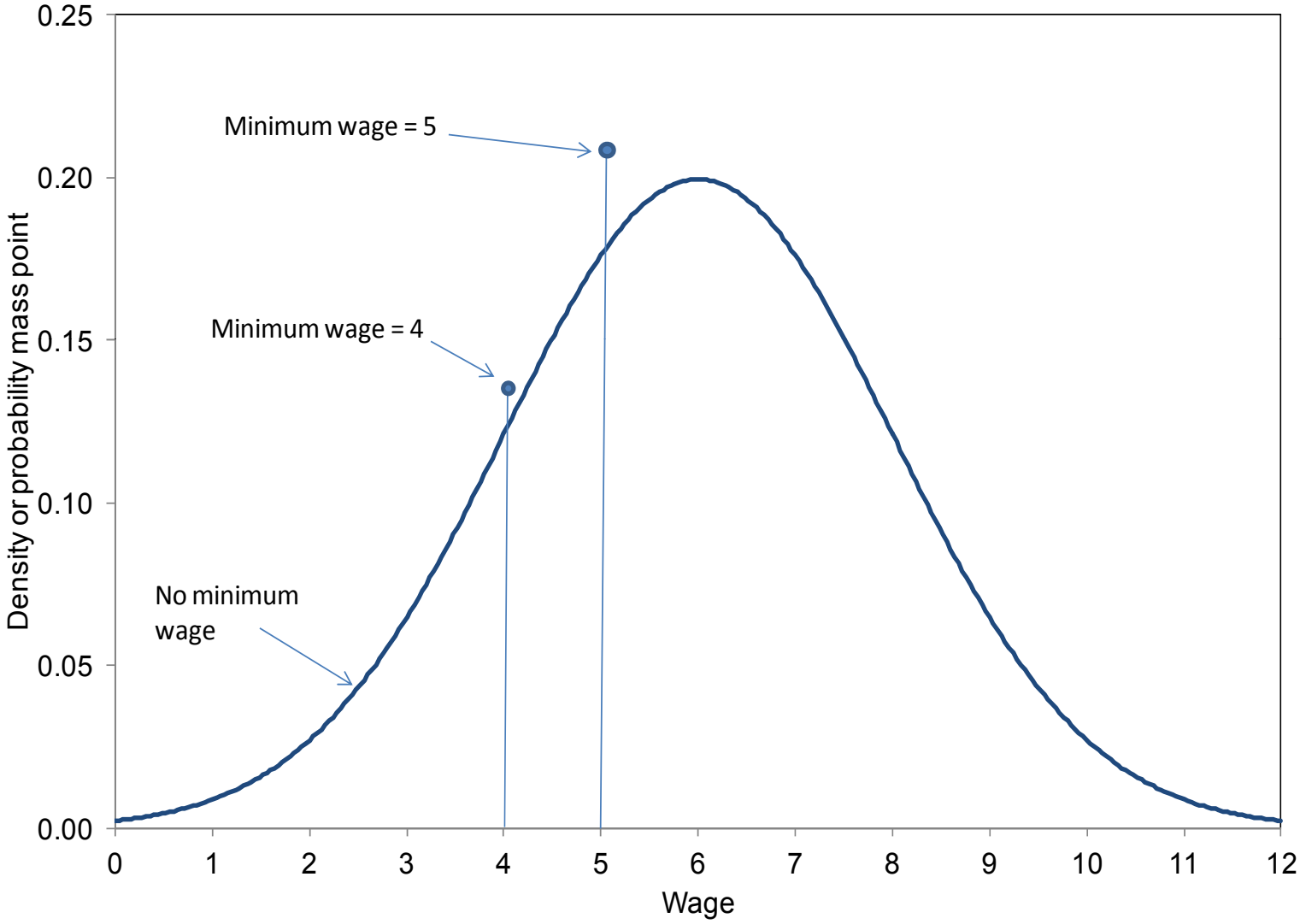
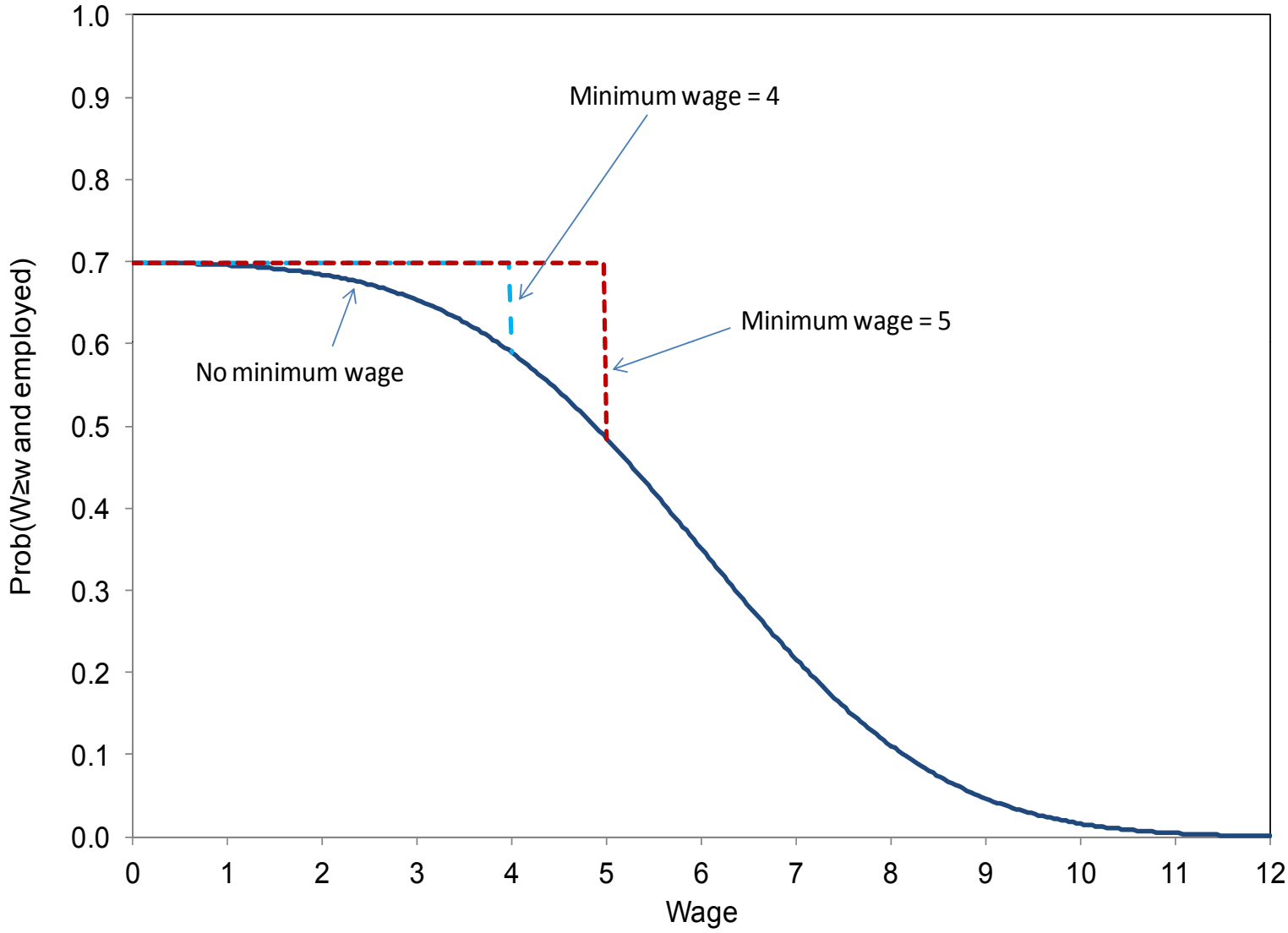


Figure 2b: Scaled survivor function for the pure spike model

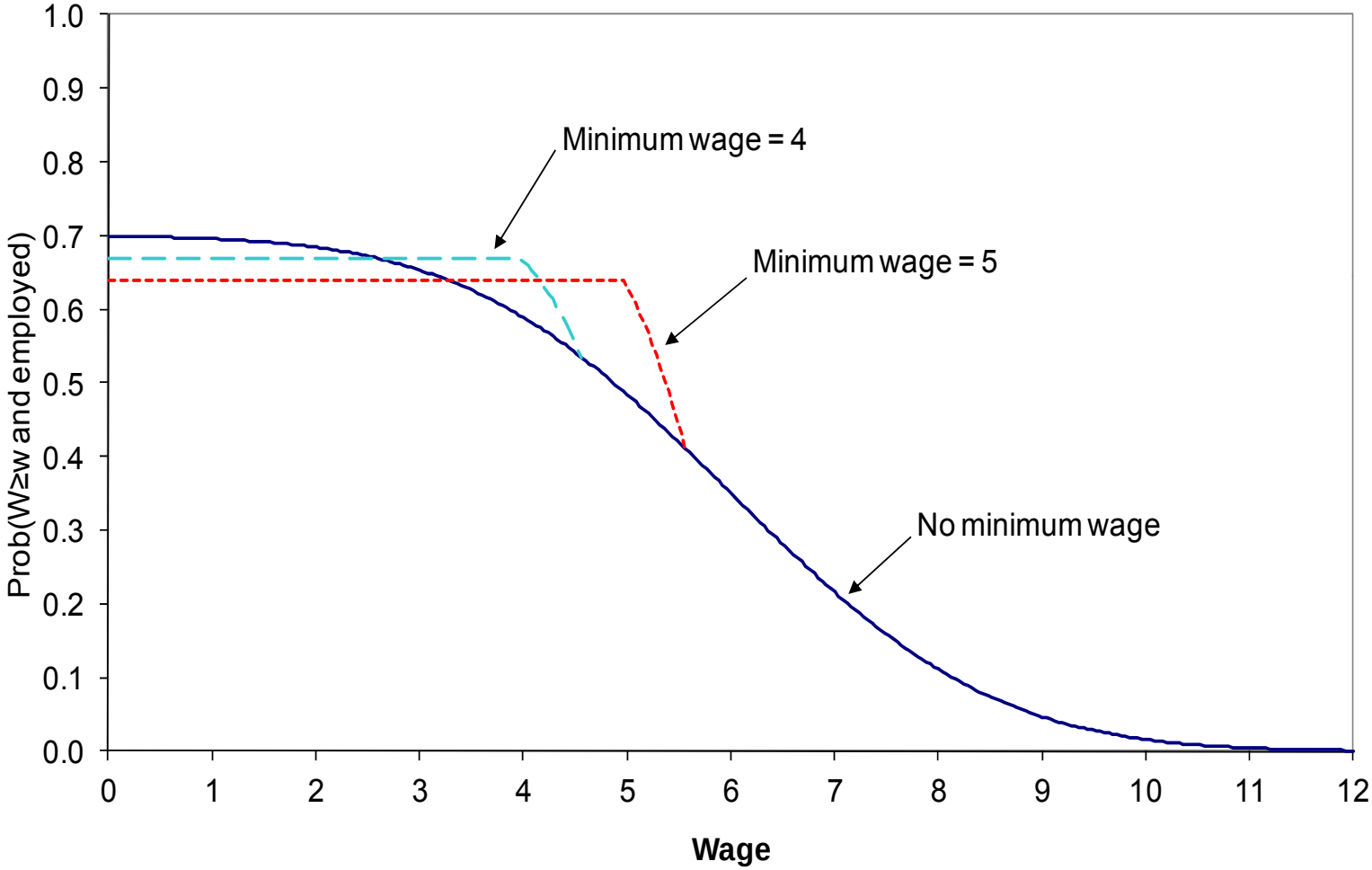


Two more general cases

- This framework can be easily generalized to a more realistic case where there is both a spike in the distribution and some disemployment effects (case 3)
- Can also add spillover effects (case 3)
- Both cases are illustrated graphically in Figures 3 and 4

Figure 3: Spike with some disemployment effects

Figure 4: Adding spillover effects



Summary of the predicted effects

- Table 1 shows the effect of an increase in the minimum wage in each of the four cases discussed earlier.
- Look at few integer values to simplify the presentation
- Correspond to the “dollar bins” that will later be used in the empirical analysis because of data limitations

Table 1a: Effect of increasing the minimum wage above $5 in a pure truncation model

	Effect of the minimum wage on the probability of being employed and earning at least:						
	0	6	7	8	9	10	11
Min. wage:							
6	-	-	0	0	0	0	0
7	--	--	-	0	0	0	0
8	---	---	--	-	0	0	0
9	----	----	---	--	-	0	0
10	-----	-----	----	---	--	-	0

Table 1b: Effect of increasing the minimum wage above $5 in a pure “spike” model

	Effect of the minimum wage on the probability of being employed and earning at least:						
	0	6	7	8	9	10	11
Min. wage:							
6	0	++	0	0	0	0	0
7	0	++	++	0	0	0	0
8	0	++	++	++	0	0	0
9	0	++	++	++	++	0	0
10	0	++	++	++	++	++	0

Table 1c: Effect of increasing the minimum wage above $5 in a “spike” model with some disemployment effects

	Effect of the minimum wage on the probability of being employed and earning at least:						
	0	6	7	8	9	10	11
Min. wage:							
6	-	++	0	0	0	0	0
7	-	+	++	0	0	0	0
8	--	+	+	++	0	0	0
9	--	-	+	+	++	0	0
10	---	-	-	+	+	++	0

Table 1d: Effect of increasing the minimum wage above $5 in a “spike” model with some disemployment and spillover effects

	Effect of the minimum wage on the probability of being employed and earning at least:						
	0	6	7	8	9	10	11
Min. wage:							
6	-	++	+	0	0	0	0
7	-	+	++	+	0	0	0
8	--	+	+	++	+	0	0
9	--	-	+	+	++	+	0
10	---	-	-	+	+	++	+

Empirical implementation

- Sample counterparts of the rescaled survivor function are computed as follows:

  (3) $RS_{pt}(w) = (1/N_{pt}) \Sigma_i 1(E_{ipt} = 1 \text{ and } W_{ipt} \geq w)$,

- E_{ipt} is a dummy indicator for employment of individual *i* in province *p* at time *t*,
- N_{pt} is the number of observations,
- W_{ipt} is the observed wage rate
- 1(.) is the indicator function.

Distribution regressions

Main estimating equation:

(4) $RS_{pt}(w) = f(W_{m,pt}, w) + \delta_p(w) + \gamma_t(w) + \beta(w)X_{pt} + \varepsilon_{pt}(w)$

- $\delta_p(w)$: province effects
- $\gamma_t(w)$: year effects
- X_{pt}: covariates (unemployment rate, province-specific trends)
- $f(W_{m,pt}, w)$: flexible function capturing the effect of the minimum wage
- A version of the equation where $RS_{pt}(w)$ is replaced with the log odds $\log[RS_{pt}(w)] - \log[1-RS_{pt}(w)]$ is later estimated.

Effect of the minimum wage

The effect of the minimum wage is captured using a set of dummy variable for each "dollar bin"

$$(5)\ f(W_m, w) = \Sigma_k \pi_k(w) D_{k,pt},$$

where $D_{k,pt}$ are a set of "dollar bin" indicator variables defined as:

$D_{k,pt} = 1$ if $k-.5 < W_{m,pt} \leq k+.5$,
$D_{k,pt} = 0$ otherwise.

The parameters $\pi_k(w)$ then capture the effect of the minimum wage relative to the base category where $W_m = W_L$. In practice, the lowest minimum wage is \$5, so W_L=5.

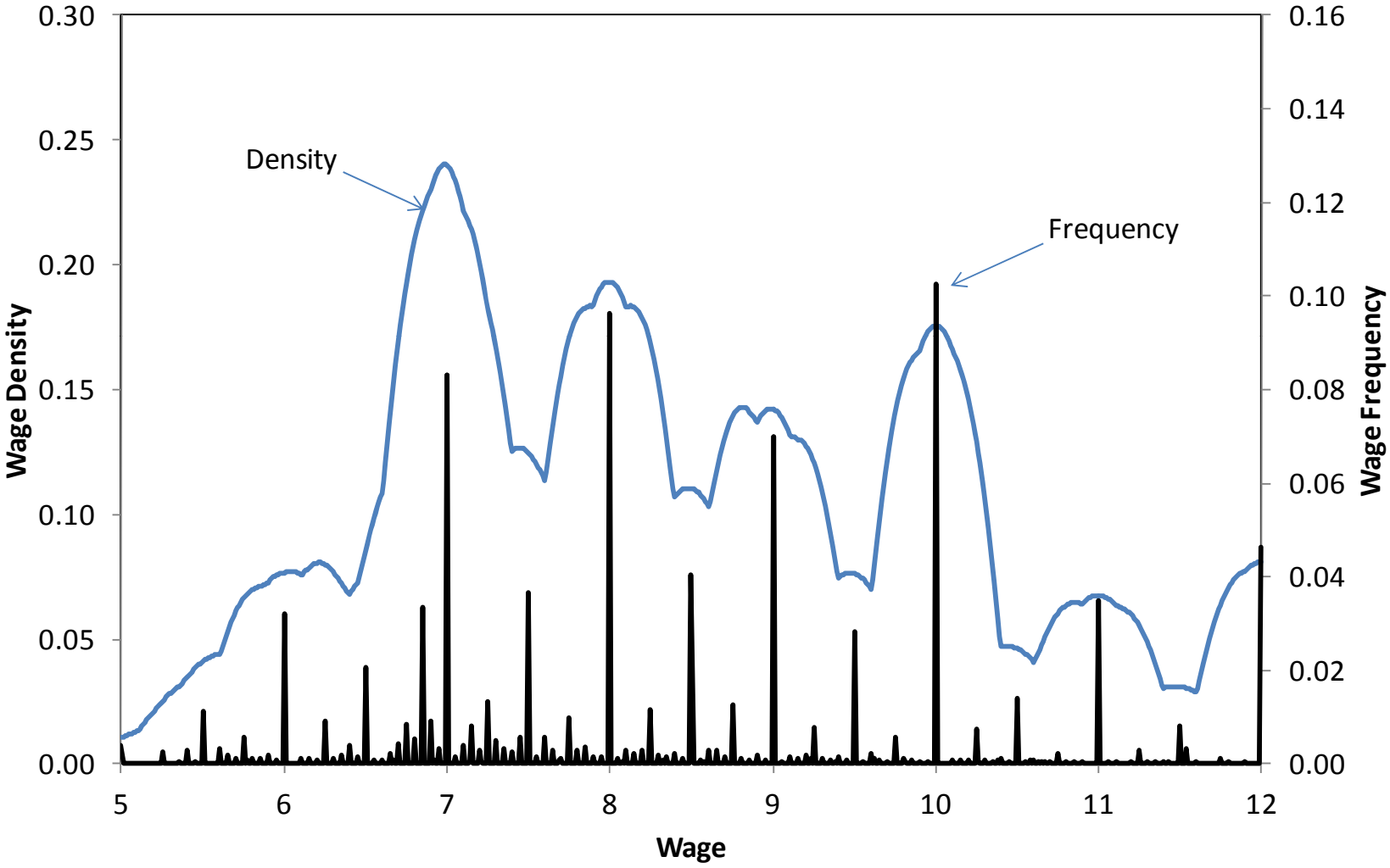
Data: Labour Force Survey

- Similar to CPS (about 100,000 adults sampled each month)
- Rotation group structure: in for 6 months, then out for good
- Questions about wages, union status, firm size, contract (permanent vs temporary) since January 1997
- Asked to incoming rotation group, updated if job changes (employer, industry, or occupation)
- Weights important since smaller provinces substantially oversampled

Wage data

- As in CPS ORG, wage asked at different periodicity depending for the convenience of respondents (hourly, etc.)
- Transformed to an hourly wage
- No wage for self-employed, so not included in analysis sample (baby sitting out...)
- The data is grouped by single dollar bins because of the extent of rounding off in the raw data

Figure 5: Wage Distribution in the 1997-2010 LFS, Workers Age 15-24



Appendix Table A1: Heaping in the LFS data

% of workers with wages rounded at the nearest:	Sample: All	Sample: Wage ~= to min wage	Sample: Wage = min wage	Predicted by uniform distribution
Dollar	47.1	49.3	31.4	1.0
50 cents	14.8	14.9	14.2	1.0
25 cents	8.6	8.1	12.5	2.0
10 cents	5.1	4.3	11.1	8.0
Total:	75.7	76.6	69.3	12.0
Other wage values:	24.3	23.4	30.7	88.0
Observations:	1474077	1295951	178126	

Notes: Data for all wage and salary workers age 15-24 from the 1997-2010

Table 2: Summary Statistics for Individuals Age 15-24, 1997-2010 LFS

	All ages	15-16	17-19	20-21	22-24
Province:					
Quebec	23.1	23.2	23.1	23.1	23.2
Ontario	38.6	39.4	38.7	38.3	38.3
Alberta	11.0	10.7	10.8	10.9	11.4
BC	13.0	12.6	12.9	13.4	13.2
Others	14.2	14.1	14.5	14.4	14.0
Education:					
Less than HS	38.7	94.2	48.6	13.5	11.0
Exactly HS	20.1	0.5	25.1	28.5	21.8
Some post-sec.	36.2	5.3	26.3	56.4	51.9
Bacc. and above	5.0	0.0	0.1	1.6	15.4

	All	15-16	17-19	20-21	22-24
Percentage:					
Female	49.0	48.1	48.8	49.0	49.8
Employed	55.9	25.7	53.3	64.8	71.5
Part-time	44.8	89.1	64.5	37.7	24.3
Unemployed	8.9	8.5	10.0	8.9	8.0
In school	49.8	80.8	61.0	39.3	26.2
At or below min wage	18.0	45.2	30.6	11.4	6.4
Average real hourly wage	10.35	7.63	8.30	10.52	12.39
Average minimum wage	7.38	7.38	7.38	7.38	7.38
Observations:	2,692,586	571,789	855,121	529,312	736,364

Minimum wages in Canada

- Few sectors (transportation, communication, etc.) under federal labor jurisdiction
- Since 1996, however, the federal minimum wage is simply the prevailing provincial minimum wage
- Lots of variation among the four largest provinces
- Other provinces more or less follow each other

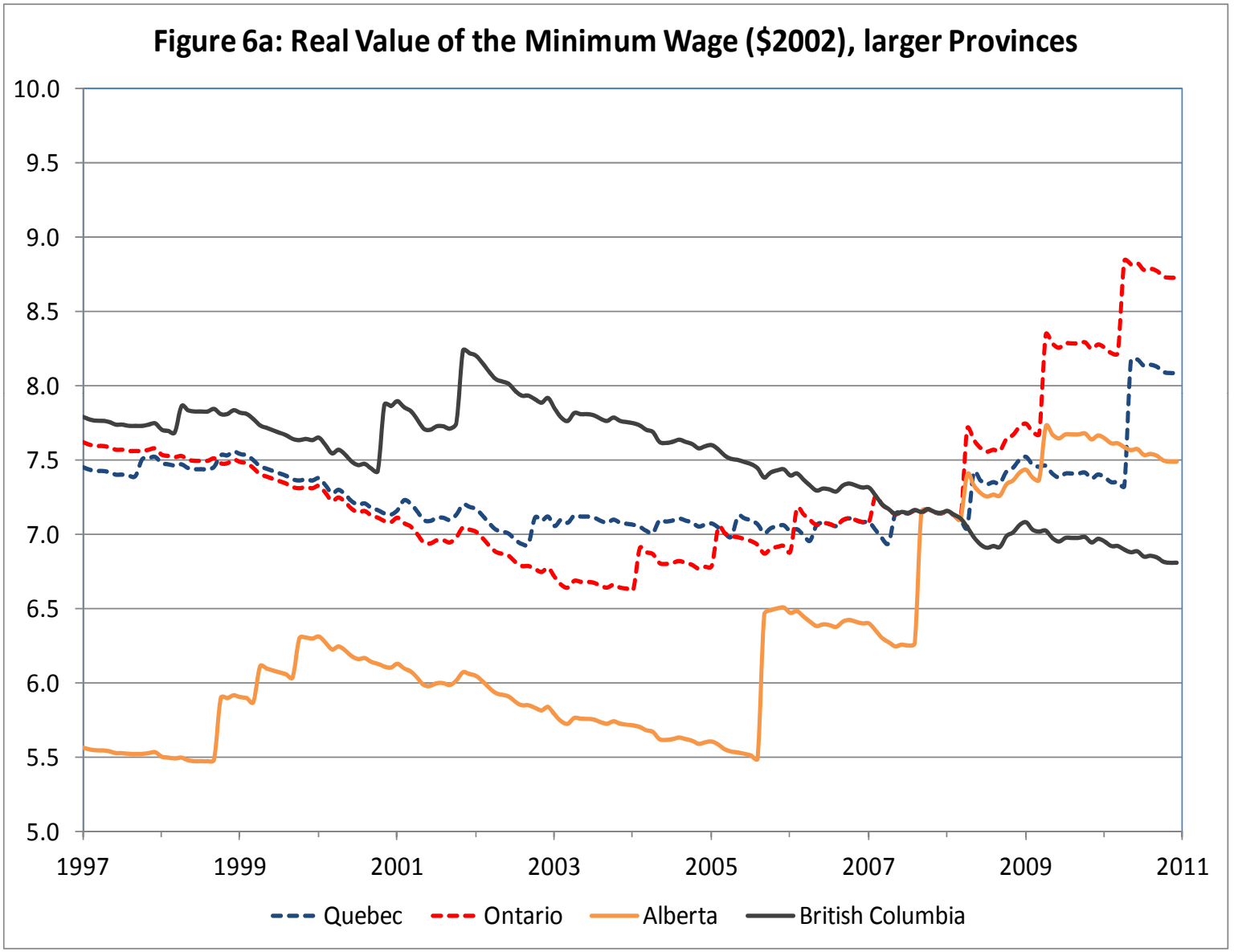


Figure 6a: Real Value of the Minimum Wage ($2002), larger Provinces
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
1997
1999
2001
2003
2005
2007
2009
2011
Quebec
Ontario
Alberta
British Columbia

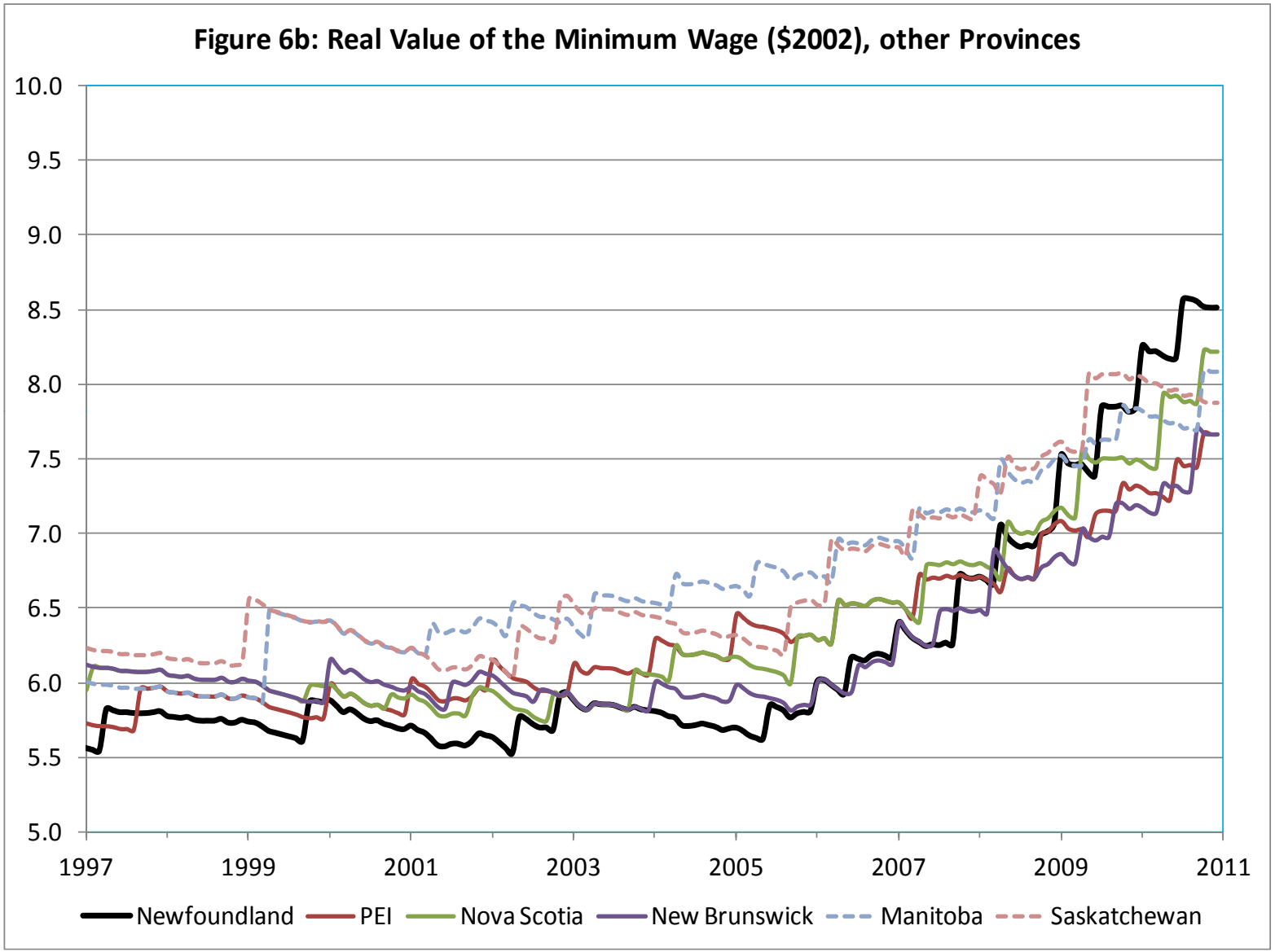


Figure 6b: Real Value of the Minimum Wage ($2002), other Provinces
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
1997
1999
2001
2003
2005
2007
2009
2011
Newfoundland
PEI
Nova Scotia
New Brunswick
Manitoba
Saskatchewan

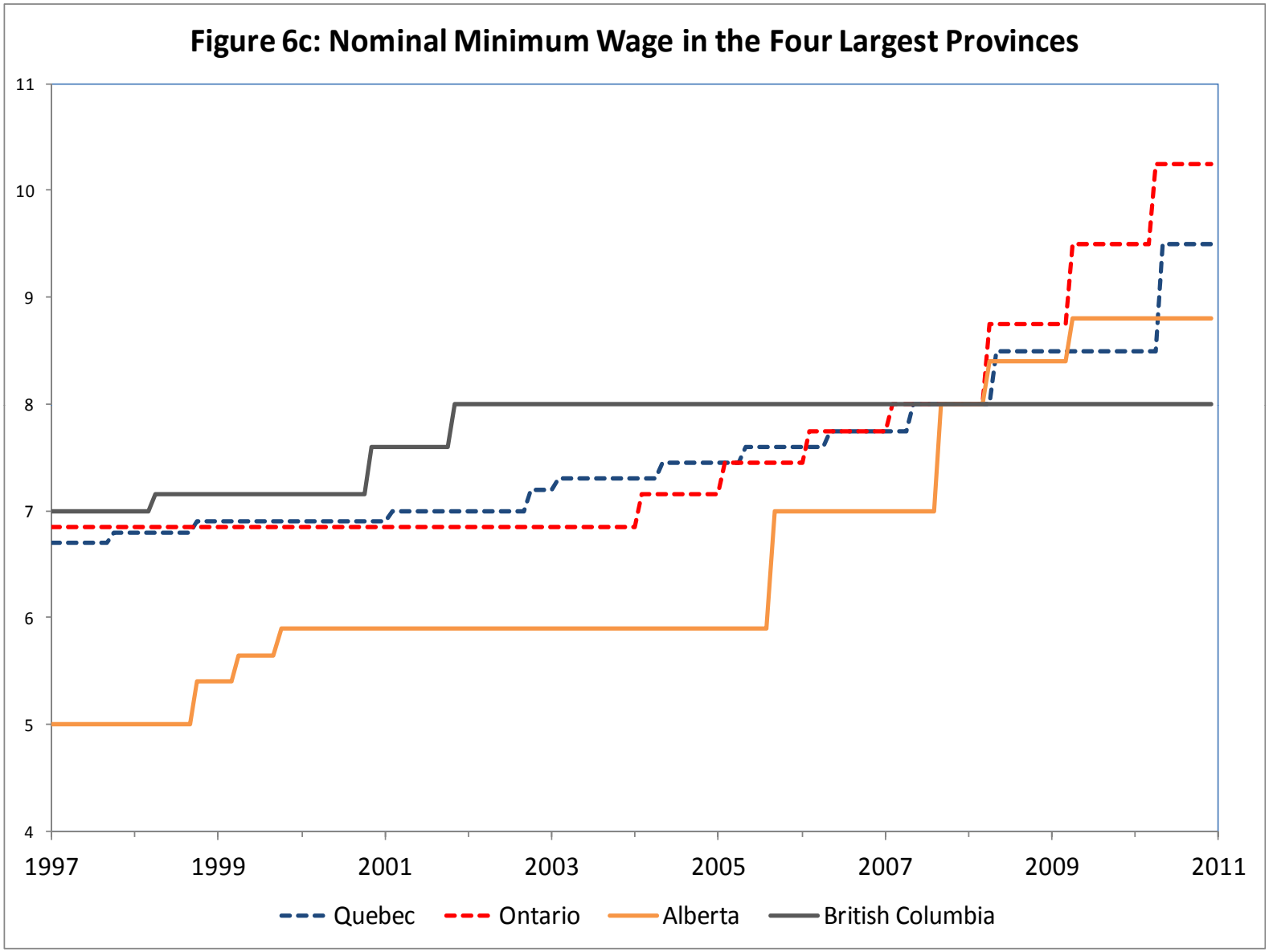
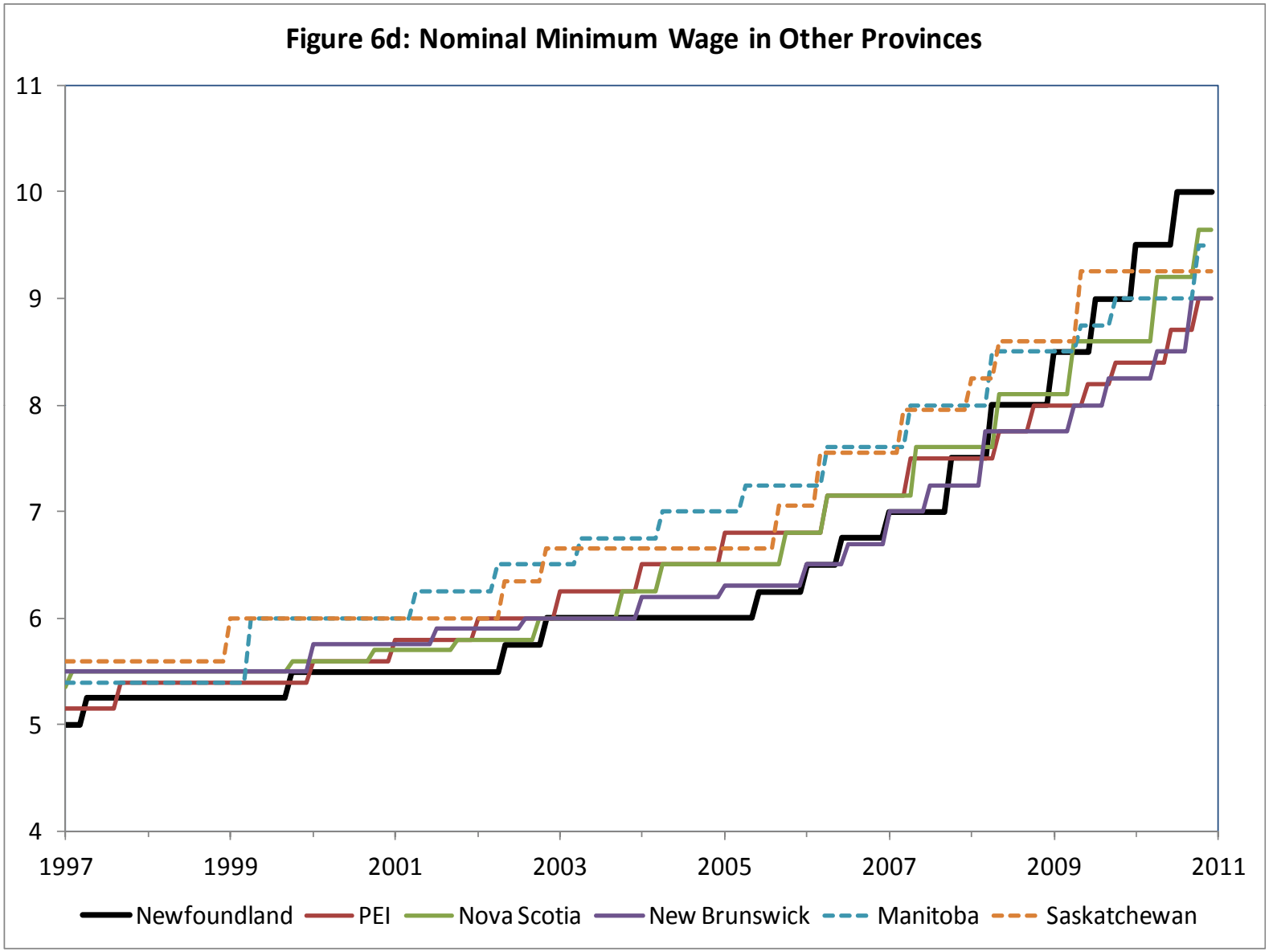


Figure 6c: Nominal Minimum Wage in the Four Largest Provinces
11
10
9
8
7
6
5
4
1997
1999
2001
2003
2005
2007
2009
2011
Quebec
Ontario
Alberta
British Columbia

Figure 6d: Nominal Minimum Wage in Other Provinces



Empirical results

- Start with standard difference-in-differences models for employment and average wages as a benchmark
  - Run separately for each age group
- Then move to distribution regressions
  - Grouped for all teenagers (15-19) and young adults (20-24) to help with precision
- Weighted using sample weights (sum of weights for province-year)
  - Small provinces have very small samples for some wage bins => efficiency issue
  - Little variation in the minimum wage for smaller provinces
- Note that the whole analysis is in nominal terms, but
  - Inflation only at 2 percent over this period
  - Year dummies are always included (look at variation across provinces in the same year)

Table 3: Traditional OLS Estimates of the Effect of the Minimum Wage on Employment, Unemployment, and Wages

	(1)		(2)		(3)		(4)	
A: Age 15-16								
Employment	-0.034		-0.015		-0.032		-0.026	
	(0.008)	*	(0.005)	*	(0.007)	*	(0.006)	*
	[0.012]	*	[0.005]	*	[0.008]	*	[0.010]	*
Unemployment	0.002		0.002		0.003		0.000	
	(0.002)		(0.002)		(0.003)		(0.002)	
	[0.005]		[0.003]		[0.003]		[0.003]	
Wage	0.798		0.892		0.716		0.748	
	(0.082)	*	(0.089)	*	(0.051)	*	(0.053)	*
	[0.172]	*	[0.187]	*	[0.038]	*	[0.059]	*

B: Age 17-19

Employment	-0.025		0.001		-0.003		0.004	
	(0.011)	*	(0.005)		(0.010)		(0.006)	
	[0.013]		[0.007]		[0.005]		[0.009]	
Unemployment	0.003		-0.002		-0.002		-0.005	
	(0.003)		(0.002)		(0.005)		(0.003)	
	[0.002]		[0.002]		[0.005]		[0.004]	
Wage	0.653		0.787		0.497		0.556	
	(0.124)	*	(0.129)	*	(0.063)	*	(0.055)	*
	[0.306]	*	[0.317]	*	[0.065]	*	[0.105]	*
Province dummies	YES		YES		YES		YES	
Year dummies	YES		YES		YES		YES	
Prov. unempl. rate	NO		YES		NO		YES	
Prov.-specific trends	NO		NO		YES		YES	

Table 3: Continuation

	(1)		(2)		(3)		(4)	
C: Age 20-21								
Employment	-0.014		0.000		-0.004		0.001	
	(0.007)		(0.004)		(0.008)		(0.006)	
	[0.005]	*	[0.003]		[0.004]		[0.003]	
Unemployment	0.008		0.003		0.008		0.005	
	(0.002)	*	(0.001)	*	(0.003)	*	(0.002)	*
	[0.002]	*	[0.001]	*	[0.004]	*	[0.002]	*
Wage	0.378		0.573		-0.060		0.021	
	(0.198)		(0.196)	*	(0.125)		(0.111)	
	[0.487]		[0.497]		[0.190]		[0.224]	

D: Age 22-24

Employment	-0.013	0.000	-0.003	0.001
	(0.005) *	(0.004)	(0.008)	(0.005)
	[0.008]	[0.008]	[0.010]	[0.012]
Unemployment	0.005	-0.003	-0.001	-0.004
	(0.003)	(0.001) *	(0.003)	(0.002) *
	[0.003]	[0.001] *	[0.004]	[0.001] *
Wage	0.402	0.611	-0.153	-0.064
	(0.228)	(0.224) *	(0.163)	(0.147)
	[0.528]	[0.524]	[0.305]	[0.320]
Province dummies	YES	YES	YES	YES
Year dummies	YES	YES	YES	YES
Prov. unempl. rate	NO	YES	NO	YES
Prov.-specific trends	NO	NO	YES	YES

Summary of the results

- Some disemployment effects for teenagers age 15-16
  - But this group is only weakly attached to the labor force (25 percent employment rate, 90 percent part-time)
- No effect for 17-19 despite clear impact of the minimum wage on their average wages
- No significant impact on either wages or employment for young adults

Distribution regression results

- Remember that the first column gives standard employment results
  - Difference here is that a set of minimum wage dummies is used instead of a linear specification
- Main diagonal plays an important role. Should be positive if some workers are “pushed up” to the new minimum (spike)
- Table 1d for the most general case with spike, disemployment effects, and spillovers is a useful reminder

Table 1d: Effect of increasing the minimum wage above $5 in a “spike” model with some disemployment and spillover effects

	Effect of the minimum wage on the probability of being employed and earning at least:						
	0	6	7	8	9	10	11
Min. wage:							
6	-	++	+	0	0	0	0
7	-	+	++	+	0	0	0
8	--	+	+	++	+	0	0
9	--	-	+	+	++	+	0
10	---	-	-	+	+	++	+

Table 4: Distribution regression Estimates of the Effect of the Minimum Wage

Teenagers age 15-16 only (marginal effects)

	Effect on the Probability of being employed and earning at least:							
	0	6	7	8	9	10	11	12
MW = 6	-0.020 *	0.074 *	0.011	-0.010	-0.012	-0.008	-0.006 *	-0.004
	(0.010)	(0.012)	(0.022)	(0.019)	(0.007)	(0.005)	(0.002)	(0.002)
MW = 7	-0.030	0.047 *	0.061	0.005	-0.002	-0.001	-0.004	-0.003
	(0.016)	(0.020)	(0.055)	(0.022)	(0.012)	(0.007)	(0.002)	(0.002)
MW = 8	-0.048 *	0.025	0.030	0.054 *	-0.006	-0.007	-0.007 *	-0.006
	(0.023)	(0.027)	(0.058)	(0.023)	(0.010)	(0.007)	(0.003)	(0.003)
MW = 9	-0.035	0.039	0.041	0.078 *	0.021	-0.002	-0.006	-0.005
	(0.021)	(0.025)	(0.063)	(0.024)	(0.020)	(0.008)	(0.003)	(0.003)
MW = 10	-0.017	0.060 *	0.079	0.115 *	0.049	0.028 *	-0.002	-0.002
	(0.014)	(0.019)	(0.057)	(0.025)	(0.032)	(0.012)	(0.003)	(0.003)
Proportion:	0.255	0.249	0.218	0.130	0.068	0.035	0.014	0.010

Table 5: Distribution regression Estimates of the Effect of the Minimum Wage:
All teenagers age 15-19 (marginal effects)

	Effect on the Probability of being employed and earning at least:							
	0	6	7	8	9	10	11	12
MW = 6	-0.010	0.133 *	0.016	-0.013	-0.015	-0.001	0.001	-0.017
	(0.012)	(0.016)	(0.033)	(0.034)	(0.018)	(0.022)	(0.017)	(0.086)
MW = 7	-0.026	0.097 *	0.096	0.013	0.009	0.025	0.011	0.042
	(0.019)	(0.027)	(0.076)	(0.041)	(0.035)	(0.031)	(0.020)	(0.102)
MW = 8	-0.057 *	0.059	0.042	0.099 *	0.002	0.014	0.004	-0.004
	(0.025)	(0.033)	(0.082)	(0.043)	(0.025)	(0.031)	(0.020)	(0.110)
MW = 9	-0.057 *	0.061 *	0.041	0.137 *	0.055	0.021	0.003	-0.013
	(0.026)	(0.032)	(0.087)	(0.043)	(0.038)	(0.033)	(0.021)	(0.110)
MW = 10	-0.053 *	0.072 *	0.075	0.200 *	0.111	0.098 *	0.011	0.006
	(0.020)	(0.026)	(0.078)	(0.047)	(0.061)	(0.039)	(0.021)	(0.099)
Proportion:	0.422	0.414	0.381	0.248	0.152	0.093	0.046	0.034

Results for teenagers

- Consistent with the model with spikes, some disemployment effects, but not spillover effects (case 3)
- “Spike effect” (coefficients on the diagonal) larger than disemployment effects (coefficients on the first column)
- “Push up” effect larger than the “push out” effects

Table 6: Distribution regression Estimates of the Effect of the Minimum Wage:
Young Adults Age 20-24 (marginal effects)

	Effect on the Probability of being employed and earning at least:							
	0	6	7	8	9	10	11	12
MW = 6	-0.021 *	0.021 *	0.018 *	-0.016	-0.026	-0.033	-0.041 *	-0.047 *
	(0.008)	(0.010)	(0.009)	(0.012)	(0.018)	(0.020)	(0.016)	(0.017)
MW = 7	-0.031	0.005	0.027	-0.015	-0.040	-0.048	-0.051	-0.056
	(0.017)	(0.015)	(0.019)	(0.018)	(0.036)	(0.036)	(0.036)	(0.037)
MW = 8	-0.045	-0.013	0.003	0.002	-0.057	-0.076	-0.084	-0.090
	(0.024)	(0.022)	(0.023)	(0.020)	(0.037)	(0.048)	(0.048)	(0.049)
MW = 9	-0.031	0.002	0.018	0.018	-0.020	-0.067	-0.085	-0.095
	(0.023)	(0.022)	(0.028)	(0.025)	(0.037)	(0.053)	(0.055)	(0.056)
MW = 10	-0.015	0.019	0.046	0.050	0.020	0.004	-0.057	-0.095
	(0.019)	(0.018)	(0.025)	(0.026)	(0.040)	(0.052)	(0.056)	(0.056)
Proportion:	0.686	0.682	0.665	0.599	0.518	0.447	0.356	0.310

Results for young adults

- No significant effects except for $6 minimum wage
- But likely a spurious effect since negative employment effect at the top end exceeds the overall employment effect (first column)

Conclusion

- Paper proposes a distribution regression approach to estimate the effect of the minimum wage on the joint distribution of wages and employment
- Results for teenagers indicates a mix of disemployment and "spike" effect. Push-up effect dominates the push-out effect.
- Underlying variation is limited (mostly 4 large provinces) which may explain some imprecision in the results
- Would be useful to apply the technique to the U.S. where state minimum wages are now playing an important role
- The proposed method could also be used in other setting where policy or other interventions have effects on employment probabilities and the earnings distribution (EITC, welfare, etc.)